

Combinatorics Participating Seminar - Coxeter Groups

Time: Friday 2-3 pm, Location: MS 5217

Spring 2026

Week 1: Introduction to Cluster Algebras

Cluster algebras were first introduced by Fomin and Zelevinsky with applications to dual canonical bases, total positivity in semisimple Lie groups, and Zamolodchikov periodicity for Y -systems of finite root systems. In this talk, we give an introduction to cluster algebras, as well as one of its first formulations in the Grassmannian related to triangulations.

Speaker: Ariana Chin

Week 2: The Basics of Coxeter Systems – Examples

Definition and some examples of Coxeter systems including Weyl groups of root systems and BN -pairs.

Speaker: Karthik Sellakumaran Latha

Reference: [BB06, §1.1 & 1.2]

Week 3: The Exchange Property of Coxeter Groups

A realization of a Coxeter group as a permutation group. Reduced words, the strong exchange property, and some consequences.

Speaker: TBA

Reference: [BB06, §1.3 - 1.5]

Week 4: The Bruhat Order of Coxeter Groups

Definition and simple examples of the Bruhat order. Basic properties of the Bruhat order. The Bruhat order on S_n and for finite Coxeter groups.

Speaker: TBA

Reference: [BB06, §2.1 - 2.3]

Week 5: Parabolic Subgroups and Quotients

Parabolic subgroups and descent classes. Bruhat order on quotients and a criterion for determining the Bruhat order given the Bruhat order on quotients.

Speaker: TBA

Reference: [BB06, §2.4 - 2.6]

Week 6: The Weak Order, Reduced Words, and the Lattice Property

Definition of the right and left weak orders, correspondence between reduced decompositions and maximal chains in an interval. The lattice property of the weak order and some consequences of the meet operation.

Speaker: TBA

Reference: [BB06, §3.1 & 3.2]

Week 7: The Word Property and Normal Forms

An algorithm for determining when two reduced decompositions represent the same word, braid moves and nil moves. Normal form, unique factorization, and some examples.

Speaker: TBA

Reference: [BB06, §3.3 & 3.4]

Week 8: Combinatorial Description of Coxeter Groups of Type B and D

Natural identifications of Coxeter groups of type B and D as certain permutation groups. Description of Coxeter generators, parabolic subgroups, and other combinatorial properties such as the Bruhat order.

Speaker: TBA

Reference: [BB06, §8.1 & 8.2]

Week 9: Additional Topics/Catch-up Time

See below for related papers. Other possible topics include: root systems, Kazhdan-Lustig and R -polynomials, cluster algebras, associahedra.

Speaker: TBA

Reference: TBA

Week 10: Additional Topics/Catch-up Time

See below for related papers. Other possible topics include: root systems, Kazhdan-Lustig and R -polynomials, cluster algebras, associahedra.

Speaker: TBA

Reference: TBA

Related Papers

Title: “On the Number of Reduced Decompositions of Elements of Coxeter Groups”

Author: Richard Stanley

Summary: Stanley gives a formula for the number of reduced decompositions of an element in the permutation group S_n , which is Weyl group of type A_{n-1} , along with some conjectures about analogies for other Coxeter groups.

Reference: [S84]

Title: “The Numbers Game and Coxeter Groups”

Author: Kimmo Eriksson

Summary: Eriksson describes how the moves of a combinatorial one-player game generates a Coxeter group, and how applications of results from Coxeter groups carry over to the generalized “node-weighted” game.

Reference: [E95]

Title: “Non-Crossing Partitions for Classical Reflection Groups”

Author: Victor Reiner

Summary: Reiner gives analogues of the lattice of non-crossing set partitions for the reflection groups of types B and D .

Reference: [Re97]

Title: “Sortable Elements and Cambrian Lattices”

Author: Nathan Reading

Summary: Reading shows that the Coxeter-sortable elements in a finite Coxeter group W are the minimal congruence-class representatives of a lattice congruence of the weak order on W .

Reference: [R07]

References

[BB06] A. Bjorner and F. Brenti. [Combinatorics of Coxeter Groups](#)

[C17] E. Chen. [Lecture Notes on Postnikov’s Topics in Combinatorics Class](#)

[E95] K. Eriksson. [The Numbers Game and Coxeter Groups](#)

[H92] J. E. Humphreys. [Reflection Groups and Coxeter Groups](#)

[R07] N. Reading. [Sortable Elements and Cambrian Lattices](#)

[Re97] V. Reiner. [Non-Crossing Partitions for Classical Reflection Groups](#)

[S84] R. P. Stanley. [On the Number of Reduced Decompositions of Elements of Coxeter Groups](#)